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THE DESIGN OF ACOUSTIC TRANSDUCER ARRAYS
USING GOAL PROGRAMMING

Susan M. Draus

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The design of acoustic transducer arrays often involves the satisfaction of multiple, conflicting design specifications. The use of conventional mathematical programming techniques in this endeavor have been inhibited by their inability to consider these multiple objectives within their design formulations. Most present techniques formulate the design problem in terms of a single objective subject to a set of design constraints which become difficult, if not impossible, to solve if these constraints are conflicting in nature. Goal Programming is an effective alternative methodology. Array pattern synthesis →

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through Goal Programming provides the optimal solution to the multiple criteria design of various classes of acoustic transducer arrays. The design problem is formulated in terms of the multiple objectives expressed by the system response functions and the desired characteristics of the design parameters. The solution of two reasonably complex planar array designs is presented. The details of their formulation and solution provide an indication of the ability of this approach to resolve even more complex array design problems.

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ABSTRACT

The design of acoustic transducer arrays often involves the satisfaction of multiple, conflicting design specifications. The use of conventional mathematical programming techniques in this endeavor have been inhibited by their inability to consider these multiple objectives within their design formulations. Most present techniques formulate the design problem in terms of a single objective subject to a set of design constraints which become difficult, if not impossible, to solve if these constraints are conflicting in nature. Goal Programming is an effective alternative methodology. Array pattern synthesis through Goal Programming provides the optimal solution to the multiple criteria design of various classes of acoustic transducer arrays. The design problems are formulated in terms of the multiple objectives expressed by the system response functions and the desired characteristics of the design parameters. The solution of two reasonably complex planar array designs is presented. The details of their formulation and solution provide an indication of the ability of this approach to resolve even more complex array design problems.

CHAPTER I

INTRODUCTION

1.1 Purpose of the Study

This study provides, through Goal Programming [3], a computerized approach to the multiple criteria design of various classes of acoustic transducer arrays. Although the study is concerned solely with the design of planar arrays, the results are indicative of the ability of this approach to resolve even more complex array design problems.

The design of acoustic arrays is often concerned with achieving a number of specific response objectives for the main beam of the array while at angles off this main beam the pattern must lie below a specified response level. Unfortunately, the achievement of both of these primary requirements is generally inhibited by their conflicting nature. For example, having achieved the desired main beam response pattern, it is relatively easy to decrease the response level of the side lobes. This is often accomplished by simply radiating a greater portion of the total input power from elements located at the array center. However, this also produces a broader beam, and therefore violates the specified response achieved by the main beam.

1.2 Statement of Specific Objectives

The specific objectives of this thesis are:

1. Develop a mathematical model which describes the multicriteria array design problem via a set of

mathematical statements and extend this model to the goal programming formulation.

2. Achieve two specific acoustic array designs using the Goal Programming approach.

3. Discuss the versatility of Goal Programming in array design relative to other tools used in such designs.

4. Through the successful nature of the results achieved and the straightforward nature of the problem formulation, encourage the use of Goal Programming in resolution of even more complex array design problems.

It should be emphasized here that the goal of this study is to develop and illustrate a design technique, not to provide specific acoustic array designs.

1.3 Introductory Definitions

Before analyzing array design, it is necessary to define several concepts. The first two definitions supply a very basic understanding of the physical characteristics involved in the problem at hand. The next three definitions provide an insight into some of the various classes of acoustic arrays in existence. Finally, the last two definitions are provided to clarify the types of mathematical models that are employed to describe the array design problem.

Transducer:

1. A transducer is a device for radiating or receiving acoustic waves. One of the chief functions of the transducer is to concentrate the radiated energy into a shaped beam which points in the desired direction in space and to suppress the radiation in other directions.

2. The radiation pattern of a transducer is a graphical representation in three dimensions of the radiation of the transducer as a function of direction. The radiation patterns of all practical transducers contain a main lobe and several auxiliary lobes called side lobes.

Arrays:

3. A linear transducer array is a set of radiating elements arranged on a line as illustrated in Figure 1.1.

4. A planar transducer array is a two-dimensional distribution of elements located in a plane as shown in Figure 1.2.

5. A spatial transducer array is a three-dimensional distribution of radiating elements as is shown in Figure 1.3.

Mathematical Models:

6. A linear model of a design problem describes all objectives to be achieved by the array elements in terms of strictly linear functions. This is often accomplished by transforming nonlinear functions to linear approximations.

7. A nonlinear model of a design problem describes at least some of the objectives to be achieved by the array elements in terms of nonlinear functions. Most array design problems fall into this category.

From these definitions, it should be apparent that there may exist a nonlinear model of a linear array, although the expression itself seems conflicting.

1.4 The Chapters to Follow

A number of planar arrays have now been designed through the Goal Programming approach with the actual calculations performed by a computer algorithm [3]. The results that have been obtained are considered highly favorable and indicative of the superiority of this approach. An analysis of selected, representative results and the models formulated to achieve them is contained in the following chapters.

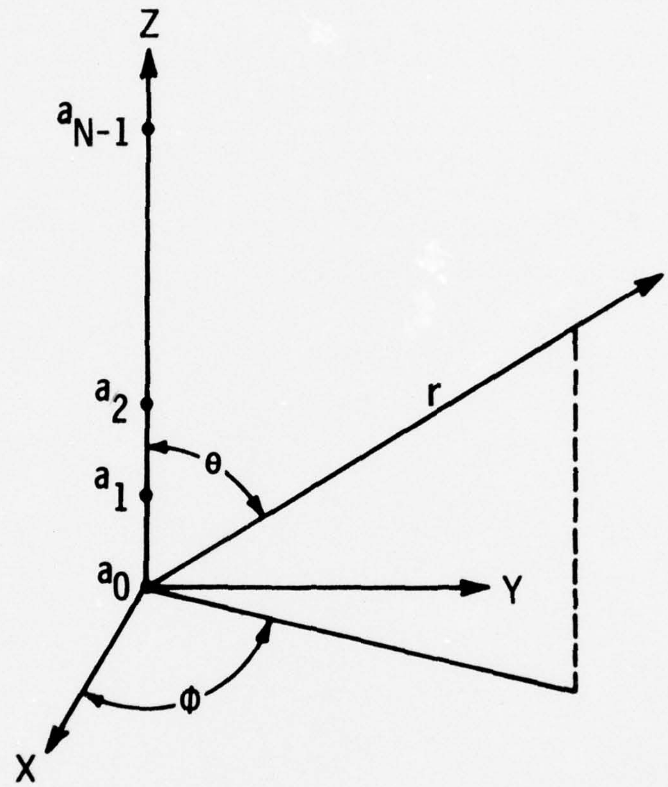

Figure 1.1. Linear Array of N Equally Spaced Elements

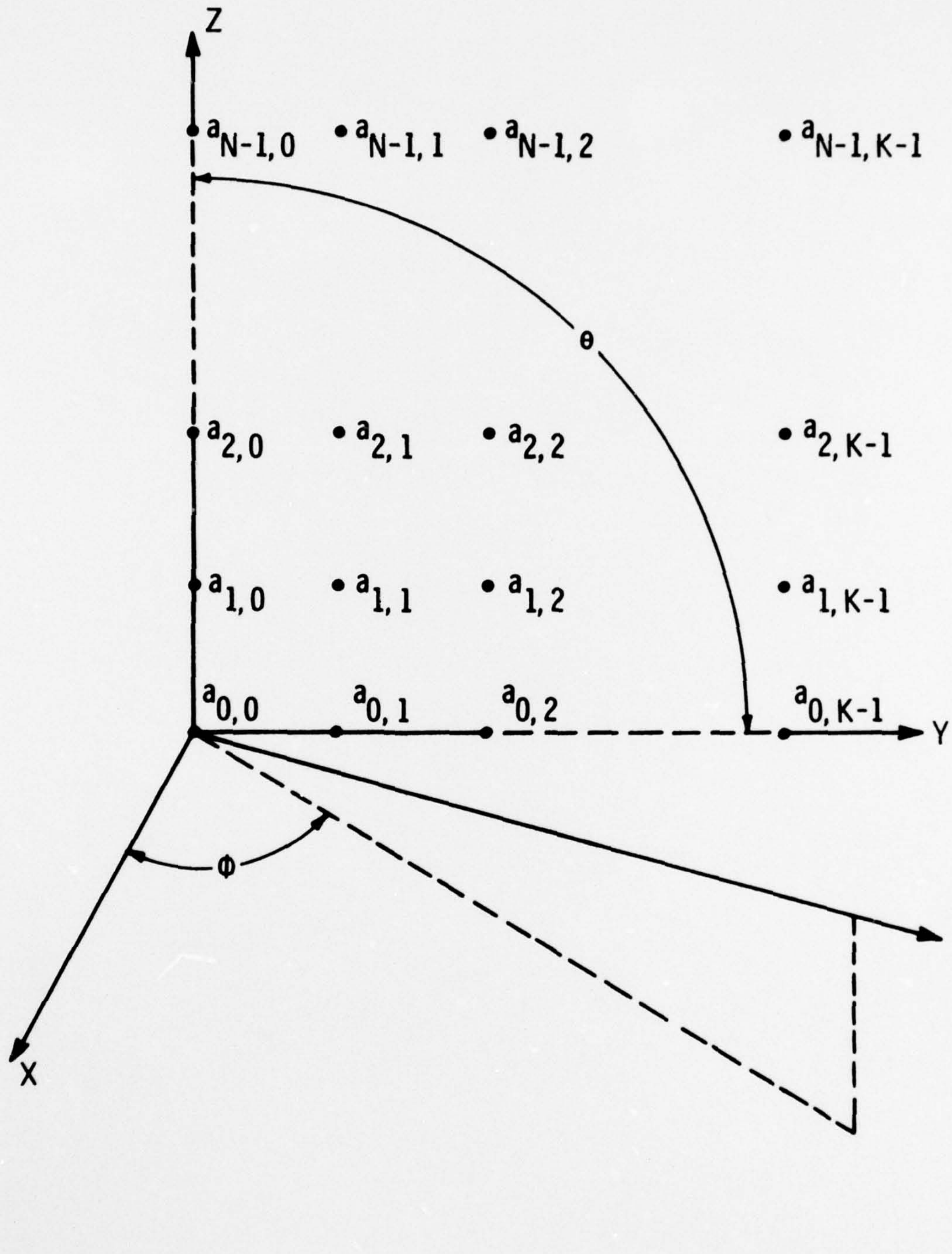

Figure 1.2. Planar Array of N x K Equally Spaced Elements

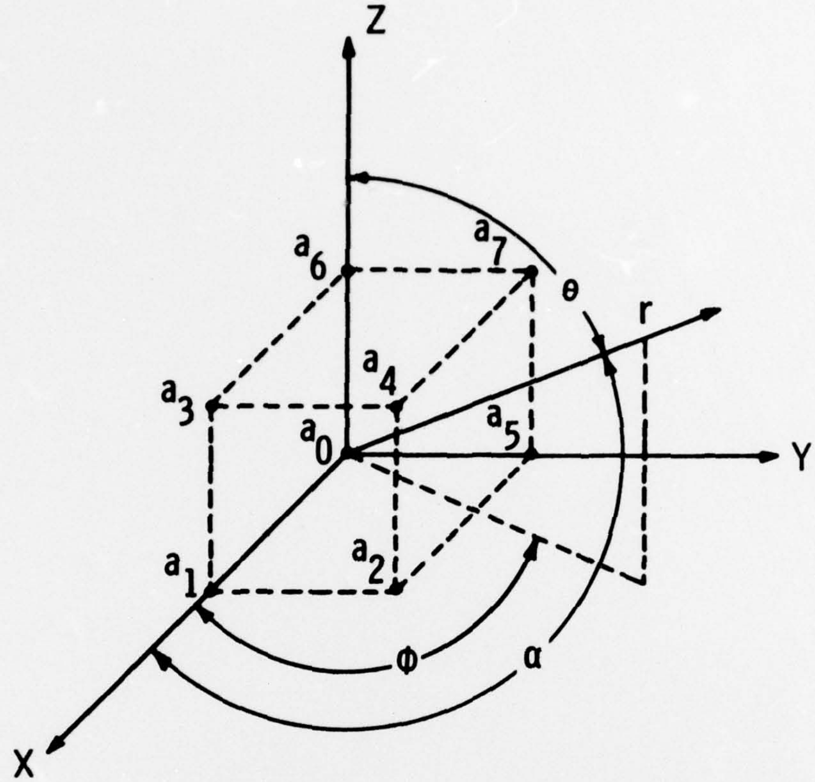

Figure 1.3. Spatial Array of 8 Equally Spaced Elements

Chapter II presents a historical perspective which will briefly introduce techniques that have been applied to acoustic array design. This chapter includes an introduction to Goal Programming and a discussion on why it was selected in view of the techniques previously or currently in use. The formulation of the mathematical model of the array design problem and its parallel Goal Programming formulation are presented in the third chapter. Finally, the results of two array design problems presented in Chapter IV provide an indication of the power and robustness of the computerized Goal Programming algorithm.

CHAPTER II

HISTORICAL PERSPECTIVE

2.1 Simulation Techniques

In the past, the chief means of array design have stemmed from studies using simulation, experiments with scale models, optimization methods, or some combination of these three. The use of simulation and scale models often provides the most costly and inconclusive results. The results are described as "inconclusive" by virtue of the fact that these techniques can offer no other criteria for the acceptance of their results than that the "optimum" design achieved was simply only the best of the relatively few examples tested.

2.2 Classical Techniques

As late as 1966, the best method for the design of linear arrays was cited as the Dolph-Chebyshev array synthesizing technique [13]. This method [9] attempts to find the array that produces a radiation pattern with the narrowest beamwidth for a specified maximum side lobe level. Whereas synthesis techniques employing Chebyshev polynomial expansions generally utilize a "minimax" criteria to provide a pattern whose deviation from the desired pattern is minimized, the Fourier integral approach to synthesis provides the best fit in the minimum-mean-square-error sense [17]. These methodologies impose several constraints on the nature of the phasings, element spacings, and excitations of the transducers which may inhibit the achievement of a superior design [13].

The Dolph-Chebyshev method appeared during 1946 at approximately the same time as the Woodward-Levinson array synthesizing technique [23]. The latter technique basically consists of reconstructing a desired array pattern from a finite number of sampled values. The process provides a pattern which achieves the desired pattern at a finite number of points, but the fit elsewhere may be quite poor. The need for a more systematic approach to design problems gave rise to the use of mathematical optimization techniques.

2.3 Mathematical Optimization Techniques

Mathematical programming as it is used in engineering, economics and other fields involves the formulation of a mathematical model of the problem and its solution via one or a combination of a number of optimization techniques. The mathematical model for the design problem at hand converts the problem into a set of mathematical statements describing the behavior of a set of response functions and any restrictions to be placed on the characteristics of the transducer array. Through the response functions, the design engineer expresses his or her objectives concerning the response to be achieved within the main beam and side lobes. This limited discourse on mathematical programming will suffice as the introduction necessary to discuss the programming techniques that have been used to resolve array design problems. A more detailed analysis of problem formulation and the process of modeling is presented in the following chapter.

The application of mathematical programming to the design of acoustical systems was little used until the 1960's. Since that time, much emphasis has been placed on the design of linear models or linear

approximations of the nonlinear case. Although the majority of array design problems will be nonlinear, linear designs have been stressed chiefly because of the inability of most optimization techniques to effectively solve a design problem with nonlinear mathematical statements.

Linear programming techniques were employed by McMahon, Hubley, and Mohammed [15] in 1972 and more recently, in early 1976, by Wilson [22] to solve for array designs which could be expressed as a strictly linear set of functions. The general linear programming problem involves optimization of a linear objective function subject to both inequality and equality constraints expressed in terms of the control variables. The problem is placed in a linear programming format and solved by the simplex method [2]. The approach of McMahon et. al. is typical of the linear programming solution techniques. Within the linear programming format, they establish a minimizing objective function which, through its cost coefficients, ranks the satisfaction of the design constraints in this order: (1) peak response constraints, (2) main beam offpeak response constraints and (3) constraints defining the desired response in the minor lobe region. Lasdon et. al. take the linear array problem one step further and apply nonlinear techniques to the design of linear arrays [13,21].

Acknowledging the nonlinear nature of most array design problems, Lasdon, Waren and Suchman focus on the study of nonlinear acoustic transducer arrays [14] and solve the nonlinear, constrained optimization problem using the Sequential Unconstrained Minimization Technique (SUMT) of Fiacco and McCormick [1]. The problem formulation in this case consists of the minimization of a linear function which is an indication of the

amount by which the solution vector fails to meet the design specifications supplied by the nonlinear constraints. The transducer design formulation then involves the minimization of a function which reflects the underachievement of the array design specifications supplied by a set of nonlinear constraints which define the desired response levels of the main beam and side lobes. Responding to the difficulty inherent in the solution of optimization problems involving nonlinear constraints, the SUMT technique uses a penalty function to convert an inequality constrained problem to a sequence of unconstrained minimization problems. If the first derivatives of this sequence of functions exist and can be easily computed from analytic expressions, then Lasdon et. al. use the descent method of Fletcher and Powell [10] for minimization. If not, then modified methods developed by Powell [16] or Stewart [18] (which solve for the minimum without calculating derivatives) are employed.

The SUMT technique in conjunction with the unrestricted methods of Powell and Stewart is not a particularly powerful one as it provides only the approximate location of the optimum, which may not be very precise when this lies in a sharp corner [1]. (Sharp ridges are typically found on the transformed surfaces created by the Interior Penalty Method used.) Unrestricted optimization techniques such as Powell's are also known to perform poorly computationally with a large number of variables [11]. In addition, the SUMT technique is not a very robust one, requiring a feasible starting point from which to begin the search for an optimum. Although there exists techniques such as those of Box, Carroll or Hensgen [1] to find a feasible base point, these often involve a computationally lengthy, iterative procedure.

Given the multimodal nature of most design problems, inherent in all nonlinear problems is the lack of assurance that a local optimum is the global value. Generally, it is necessary to start from different points in pursuit of the global value and in order to avoid acceptance of a poor local optimum. However, with the extensive set of restrictions generally present in array design problems, it is difficult to obtain valid alternative starting points which satisfy all the constraints and are significantly different.

Avoiding the need for an initial feasible solution, H. S. C. Wang employed the method of Lagrange multiplier with main beam normalization as a single constraint in an attempt to minimize the side lobe power [19]. Assuming that the output power is a quadratic of the amplitude shading coefficients, this method leads to a set of linear simultaneous equations which he solves using a standard numerical technique. Essential in the use of this method is the existence of gradients that can be found directly from analytical expressions.

The method of Lagrangian multipliers is not commonly used in optimizing very complex systems as the required calculations become intolerable. Further, the final product of such calculations is only a stationary point and it must be evaluated to guarantee that it is an optimum [1].

In more recent work, Wang uses the method of steepest ascent in pattern synthesis [20]. His problem formulation consists of objective functions which are "suitable" linear combinations of the acoustical power radiated. The development of the mathematical model of the problem is an iterative procedure with the "suitable" linear combinations of the

response functions in the objectives derived from a number of attempts at the "appropriate" combinations. Solving for these objective functions, Wang employs the method of steepest ascent to search for the characteristics of the array which give a maximum value of an objective function.

The method of steepest ascent is basically a cyclic search technique which approximates the line of steepest ascent. At each point in the search, the local function gradient is evaluated and search for the restricted optimum is made along the resulting direction of steepest ascent. Although each one-dimensional search is searching in the "best" direction, this method is not very effective. It can easily be seen that the direction of steepest ascent as it is defined here is only a local property which does not account for interactions between variables [1]. Computationally, the technique is cumbersome since the derivatives of the objective function must be evaluated at each base point in the search. As in the method of Lagrange multipliers, the existence of easily established gradients is critical in the use of this technique. If the gradients cannot be found directly, they must be estimated by local explorations which may be quite excessive.

2.4 Goal Programming

Since unrestricted methods of optimization are generally better than the constrained optimization techniques [1], techniques such as SUMT have focused on the replacement of a constrained optimization formulation by an equivalent problem without constraints. Goal Programming [3], a tool for multicriteria design, was developed by Charnes and Cooper for the strictly linear problem in the 1950's [8] and has since been expanded by Ignizio [3] to consider the nonlinear model as well. The Goal

Programming problem formulation expresses the array design problem in terms of its multiple, conflicting objectives (such as the desired main beam and side lobe response levels). The general formulation establishes the problem as one of minimizing the difference between the pattern achieved by the antenna array design and the desired response levels expressed in the objectives. The specific Goal Programming method used in this thesis to achieve the optimum array design is a modification, by Ignizio, of the numerical search technique of Hooke and Jeeves [12]. This method, also known as modified pattern search, begins with an initial base point and perturbs about this point along all coordinates of the array in search of an improved solution in terms of the prioritized design objectives. The Hooke and Jeeves method is referred to as an accelerated search technique [1] as it evaluates the success along any direction and accelerates the ongoing search along successful paths and reduces the distance explored in the event of failure to improve the design. Further discussion of the modified pattern search algorithm can be found in Appendix B.

The modified Hooke and Jeeves optimization technique was employed in lieu of a number of numerical search techniques as it is a simple and effective method of minimization which is able to handle a large number of variables and multiple objectives and possesses a range of convergence criteria. Numerical search methods require only the evaluation of the achievement function at a particular location, whereas gradient techniques require both function and gradient values to be found at each location; if the gradient cannot be found directly from an analytical expression, it must be estimated by local exploration demanding several experiments.

Therefore, in light of the size of most design problems, pattern search is much more efficient in terms of computation time.

Optimization using nonlinear Goal Programming does not require a feasible starting point, one satisfying all objectives, as do techniques such as SUMT and Complex [1], removing the initial task of seeking what may be a nonexistent point. Therefore, this method is more efficient in its search for the global optimum through its ease in choosing significantly different starting points. However, as in all methods of nonlinear optimization, there is no known way to guarantee that the solution or system design is the global optimum. For these reasons, the modified pattern search method of nonlinear Goal Programming was selected as the tool for acoustic array design in this study.

CHAPTER III

PROBLEM FORMULATION

3.1 Mathematical Model Formulation (Planar Arrays)

The initial step in the Goal Programming formulation of array design is the development of a preliminary mathematical model which essentially describes the multicriteria design problem via a set of mathematical statements. This introduction to mathematical models of planar arrays will be followed by the extension of the preliminary mathematical model to the Goal Programming formulation. A specific planar array design model formulation will be used to illustrate the process.

All planar array design problems studied were symmetric about the horizontal and vertical axes in the plane and uniformly spaced at one-half wavelength apart. However, after reviewing the Goal Programming formulation which follows, it should be apparent that this approach can easily be extended to more complex array design problems.

A primary goal of planar array design is to form a certain radiation pattern. The associated design objectives are expressed in terms of a response function or set of response functions which describe the gain or directivity of the array. This gain is measured over the standard θ,ϕ coordinate system (in which θ is the angle of incidence as measured from the positive Z axis and ϕ is the roll angle from the X axis of the projection in the X-Y plane, (see Figure 3.1) and will be

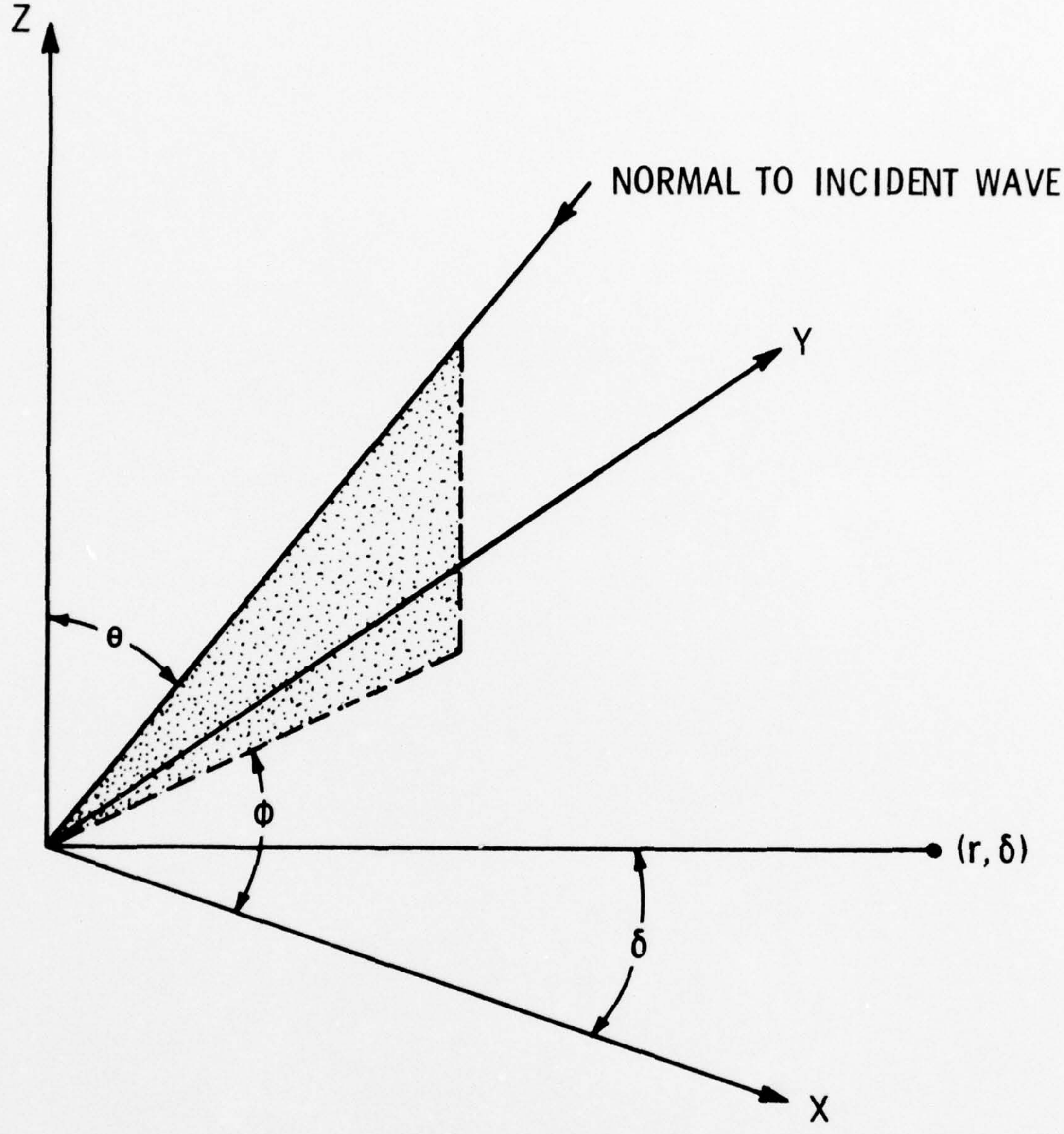

Figure 3.1. Coordinate System

(r,δ) are the Polar Coordinates of a Transducer Element in the X-Y Plane

denoted as $g(\theta,\phi)$. Each specific value of (θ,ϕ) considered in the objectives is thus assigned a design response, $g_i(\theta,\phi)$.

The response objectives of the particular design problems studied were to obtain a main beam with uniform response and to have low response elsewhere. Defining index sets I_A and I_B such that I_A contains those values of i for which $g_i(\theta,\phi)$ must be uniform at a specified level γ and I_B those for which $g_i(\theta,\phi)$ is to lie below another level β , then the design objectives may be expressed as:

$$g_i(\theta,\phi) = \gamma \qquad i \in I_A \tag{3.1}$$

and

$$g_i(\theta,\phi) \leq \beta \qquad i \in I_B \ . \tag{3.2}$$

Elaborating on the response functions, the power response objectives of a symmetric planar array follow (see Appendix A for the derivation of these functions from the general case):

$$g_i(\theta,\phi) = 10 \log\left[16\left[\left(\sum_{m=1}^{M} A_m \cdot \cos\alpha_m \cdot \cos B_m \cdot \cos C_m\right)^2 + \left(\sum_{m=1}^{M} A_m \cdot \sin\alpha_m \cdot \cos B_m \cdot \cos C_m\right)^2\right]\right] = \gamma$$

$$\text{where } i \in I_A \tag{3.3}$$

$$g_i(\theta,\phi) = 10 \log\left[16\left[\left(\sum_{m=1}^{M} A_m \cdot \cos\alpha_m \cdot \cos B_m \cdot \cos C_m\right)^2 + \left(\sum_{m=1}^{M} A_m \cdot \sin\alpha_m \cdot \cos B_m \cdot \cos C_m\right)^2\right]\right] \leq \beta$$

$$\text{where } i \in I_B \tag{3.4}$$

given that:

$$B_m = \frac{2\pi}{\lambda} (X_m \cdot \sin\theta \cdot \cos\phi) \tag{3.5}$$

$$C_m = \frac{2\pi}{\lambda} (Y_m \cdot \sin\theta \cdot \sin\phi) \tag{3.6}$$

and

γ = the desired response level for the main beam,

β = the desired response level for the side lobes,

I_A = the set of responses pertaining to the main beam,

I_B = the set of responses pertaining to the side lobes,

M = the number of sets (of four) array elements in quadrantal symmetry,

λ = the operating wavelength of the array,

X_m, Y_m = the cartesian coordinates (in wavelengths) of the m^{th} set of four elements,

α_m = the phase angle associated with the m^{th} set of four elements,

and

A_m = the shading factor of the m^{th} set of elements.

The particular design problems studied involved obtaining a main beam with uniform response, within 3 dB if possible, and a low response elsewhere (a goal of -20 dB was set). The control variables which were permitted to be regulated to achieve the specified response levels, were either the phasing of the array elements, their amplitude or both. Therefore, the design problem becomes that of finding the vector $\bar{\alpha}$, $\bar{A}$, or (α, A) best satisfying Equations (3.3) and (3.4). Due to the symmetry inherent in the problem, it is only necessary to specify the elements of a single quadrant. The design objectives can now be restated as:

$$g_i(\theta,\phi) - g(0°,0°) = 0 \text{ dB} \qquad i \in I'_A \tag{3.7}$$

and

$$g_i(\theta,\phi) - g(0°,0°) \leq -20 \text{ dB} \qquad i \in I'_B \ . \tag{3.8}$$

Each design objective of Equations (3.7) and (3.8) corresponds to the desired radiation level at some value of θ and ϕ. The size of I'_A and I'_B, the number of objectives constraining the main beam and side lobe levels respectively, is determined by the number of values of θ and ϕ that must be considered so as to attain the desired pattern. Oftentimes, if the range between values of θ and ϕ defined in the model's objectives is "too large" (implying that a small number of radiation levels have been included in the model), the pattern will attain the desired level within the constrained area, but with violations of the desired response occurring within those regions not specified in the objectives. On the other hand, it is essential to keep the number of objectives at a minimum as computation time and computer storage requirements rapidly increase as the number of objectives increases.

Due to the symmetry of the models considered, the range on θ and ϕ included in the response objectives can be reduced to extremes of $0°$ and $90°$ (i.e., the first quadrant). It is important to incorporate into the model any properties of the design that can reduce the inherent size of the problem.

3.2 Nonlinear Goal Programming Model Formulation

The second phase of the problem formulation involves the extension of the preliminary mathematical model to the goal programming formulation. This involves the addition of deviation variables in the objective

functions and the statement of the "achievement function" in terms of these variables.

On the left-hand side of the goal programming formulation, each objective function will contain a negative and positive deviation variable (expressed as n_i and p_i respectively). The value of n_i reflects the negative deviation from the desired response level for any specific solution vector, while the value of p_i reflects the positive deviation. Letting b_i equal the desired response level for a particular $g_i(\theta,\phi)$, every response objective function can be expressed as an equality of the form:

$$g_i(\theta,\phi) + n_i - p_i = b_i \ .$$

Looking at the side lobe objectives for clarification of the role of the deviation variables, if the radiation level of a particular $g_i(\theta,\phi)$ is below that required 20 dB dropoff from the main beam level, the negative deviation variable (n_i) will take on the value of the additional difference. Conversely, if the value of the response function is greater than the design specifications, the positive deviation variable (p_i) will assume the value of the design violation.

An expression of the design objectives in terms of the deviation variables follows:

$$g_i(\theta,\phi) - g(0°, 0°) + n_i - p_i = 0 \text{ dB} \qquad i \in I'_A \qquad (3.9)$$

and

$$g_i(\theta,\phi) - g(0°, 0°) + n_i - p_i = -20 \text{ dB} \qquad i \in I'_B \ . \qquad (3.10)$$

A key assumption of nonlinear Goal Programming is that the design engineer can establish preemptive priorities [4] for each objective or group of objectives. The first priority is always assigned to any absolute objective(s), i.e., any objective(s) whose satisfaction is absolutely necessary for the resulting solution to be implementable. Next, priorities are associated with all remaining, nonabsolute objectives.

In solution, nonlinear Goal Programming initially satisfies, as best as is possible, the objectives with the highest priority. Continuing on to the next priority level, the objectives at that level are satisfied as closely as is permissible without degrading the achievement of any higher priority objectives. This process continues until all priority levels have been considered and resolved as well as possible. It can be seen from this process that objectives at a higher priority level are immeasurably more important than those of a lower level.

Judgmental or inherent weights may also be assigned to the commeasurable objectives or goals within a priority level. These weights must be nonnegative numbers. When trying to achieve a difficult response pattern in antenna design, the weighting factors may be assigned in such a way that the algorithm will strive to achieve the desired pattern within an area of resistance at the expense of the remaining response. The determination of the "correct" weighting factors for a given design is often an iterative process with completion of the process only upon arrival at a "satisfactory" antenna array design.

In order to remove any ambiguity concerning the deviation variables in the forthcoming achievement function, it is necessary to present another restatement of the problem formulation:

$$g_i(\theta,\phi) - g(0°,0°) + n_i - p_i = 0 \text{ dB} \qquad i \in I'_A \qquad (3.11)$$

and

$$g_j(\theta,\phi) - g(0°,0°) + n_j - p_j = -20 \text{ dB} \qquad j \in I'_B \ . \qquad (3.12)$$

Therefore, the single goal of the design problem as defined, without any restrictions on the range of the phasing or shading coefficients, is to minimize the quantity $(n_i + p_i)$, which demands a uniform response in the main beam, and p_j , which requires a response in the side lobes less than or equal to a level 20 dB below that of the main beam. This goal is expressed:

$$g_1(\overline{n},\overline{p}) = (n_i + p_i + p_j) \qquad i \in I'_A \ , \quad j \in I'_B \ .$$

The formulation of the achievement function is the final step in the statement of the Goal Programming model. The achievement function is a linear function of the deviation variables which will always be minimized. In order to maintain the preemptive priorities of each goal, the achievement function is always an ordered vector of the form [3]:

$$\overline{a} = \{g_1(\overline{n},\overline{p}),\ g_2(\overline{n},\overline{p}),\ .\ .\ .,\ g_K(\overline{n},\overline{p})\} \ ,$$

where

(1) $g_k(\overline{n},\overline{p})$ is a linear function of the deviation variables,

(2) k is the priority level associated with the objectives expressed in $g_k(\overline{n},\overline{p})$,

(3) the dimension of $\overline{a}$ represents the number (K) of preemptive priority levels among the objectives,

(4) the number of preemptive priorities are equal to or less than the total number of objectives,

and (5) $\overline{a}$ will always be minimized.

The achievement function for the design problem formulated to this point is:

$$\text{minimize} \quad \bar{a} = \{g_1(\bar{n}, \bar{p})\}$$

or

$$\text{minimize} \quad \bar{a} = \{(n_i + p_i + p_j)\} \quad .$$

This achievement function assumes equally weighted priorities. If a weight of w_i were associated with each of the objectives of the main beam, $g_1(\bar{n}, \bar{p})$ would read $[w_i(n_i + p_i) + p_j]$.

In final solution, the achievement vector will be equivalent to $\bar{0}$ if all of the objectives were met and will take on positive values if all could not be met. The solution of a design model is considered optimal if the corresponding value of $\bar{a}$ is the same or preferred to the value of $\bar{a}$ for any other feasible solution.

The final, completed formulation of the planar array design model follows:

Find the solution vector (phasings, amplitudes or both) so as to minimize:

$$\bar{a} = \{(n_i + p_i + p_j)\} \quad ,$$

such that:

$$g_i(\theta,\phi) - g(0°,0°) + n_i - p_i = 0 \text{ dB} \qquad i \in I_A' \tag{3.13}$$

$$g_j(\theta,\phi) - g(0°,0°) + n_j - p_j = -20 \text{ dB} \qquad j \in I_B' \tag{3.14}$$

and

$$\bar{n}, \bar{p} \geq 0 \quad .$$

We now introduce the formulation of a specific antenna design problem. The problem to be analyzed is the design of a 12 x 12 circular, symmetric, uniformly spaced planar array. The square grid has the corners truncated so that the array approximates a circular aperture. In this analysis both the amplitude and phasing will be variable. Therefore, the example to be analyzed seeks the optimum phasing and shading coefficients of the 104 element array. Due to the symmetry inherent in the problem, it is necessary to consider only four sets of 26 elements. (Figure 3.2 indicates the location of the 26 elements in the first quadrant of the X-Y plane.)

The pattern to be achieved is a circular 30° diameter beam. Thus, the design specifications require a circular 30° beam with uniform response and all angles off the main beam at a level 20 dB below the main beam response. In addition to these (ideal) design objectives, it is also desirable to constrain the characteristics of the array elements to allow for easy implementation. The phase angles are defined within the range of 0° to 360° and the shading coefficients are permitted to take on only positive values.

The formulation of the goal programming model established to satisfy these design specifications follows:

Find the vector $(A_1, A_2, \ldots, A_{26}, \alpha_1, \alpha_2, \ldots, \alpha_{26})$ so as to minimize:

$$\bar{a} = \{(n_1 + n_2 + \ldots + n_{26} + p_{27} + \ldots + p_{52} + n_{53} + \ldots + n_{72}),\ (n_i + p_i + p_j)\} \quad ,$$

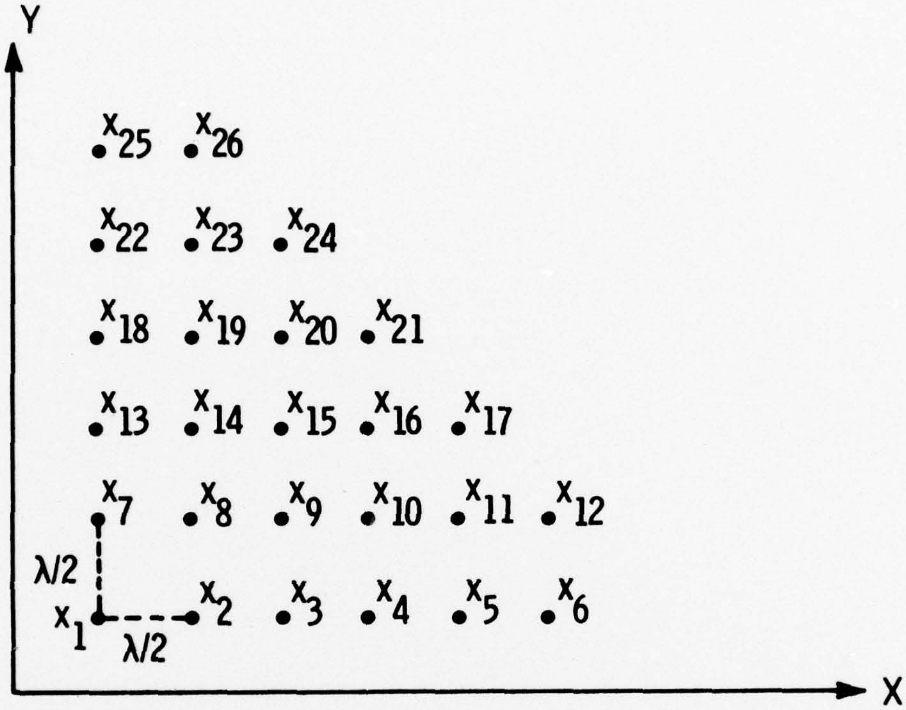

Figure 3.2. Array Elements in the First Quadrant of the X-Y Plane

such that:

$$\alpha_m + n_k - p_k = 0° \qquad k = 1,2, \ldots, 26, \tag{3.16}$$

$$\alpha_m + n_l - p_l = 360° \qquad l = 27, \ldots, 52, \tag{3.17}$$

$$A_m + n_n - p_n = 0 \qquad n = 53, \ldots, 72, \tag{3.18}$$

$$m = 1,2, \ldots, 26$$

$$g_i(\theta,\phi) - g(0°,0°) + n_i - p_i = 0 \text{ dB} \qquad i \in I_A{}' \tag{3.19}$$

with theta ranging from 0° to 15° for values of phi from 0° to 90°, and

$$g_j(\theta,\phi) - g(0°,0°) + n_j - p_j = -20 \text{ dB} \qquad j \in I_B{}' \tag{3.20}$$

for theta ranging from 16° to 90° at all values of phi from 0° to 90°, where:

$$g_i(\theta,\phi) = 10 \log \left[16[(\sum_{m=1}^{26} A_m \cdot \cos \alpha_m \cdot \cos B_m \cdot \cos C_m)^2 + (\sum_{m=1}^{26} A_m \cdot \sin \alpha_m \cdot \cos B_m \cdot \cos C_m)^2]\right]$$

and $\bar{n}, \bar{p} \geq 0$.

Again,

$$B_m = \frac{2\pi}{\lambda}(x_m \cdot \sin\theta \cdot \cos\phi)$$

and

$$C_m = \frac{2\pi}{\lambda}(y_m \cdot \sin\theta \cdot \sin\phi) \quad .$$

Note that:

1. The size of I_A' and I_B' is dependent upon the size of the sampling of radiation levels necessary to attain the desired pattern without response violations within the undefined regions.

2. The achievement vector as it is written has equally weighted objectives which assumes the design achievement will not be confronted with resistance in any particular area. This will seldom, if ever, be the case in practice.

3. The operating wavelength, λ , for all of the design problems is assumed to be 1.

4. The cartesian coordinates (x_m, y_m) are expressed in intervals of 1/2 wavelength.

The subroutine (YVALUE) required to be supplied by the user of the general computerized Goal Programming code to define this particular model formulation for the optimization algorithm is presented in Appendix C. In addition, Appendix B provides further explanation on Goal Programming and the design problem formulation.

In the next chapter, we present the results of the application of the nonlinear Goal Programming code to both this problem (i.e., a circular beam) and also to the problem of forming a rectangular beam. Although numerous other problems were solved, these two problems are typical of the approach and results obtained.

CHAPTER IV

EMPIRICAL RESULTS

4.1 Introduction

The solutions to all array designs discussed herein were obtained using the nonlinear Goal Programming code as listed and documented in the appendix of Reference 3. All computer analyses were performed by an IBM 370.

The 12 x 12 array designs presented herein are the result of a three stage experiment. Initially, the design problems were formulated to solve for a number of 6 x 6 planar arrays. The study was then extended to 8 x 8 planar arrays. Finally, the 12 x 12 models were resolved.

The study began with the smaller arrays for several reasons. First, the relatively small solution times permitted extensive manipulation of the parameters of the goal programming formulation (weighting factors, number of response objectives, etc.) at a far smaller cost (the computation time at each iteration of the 12 x 12 array problem was approximately five times greater than its 6 x 6 counterpart). These manipulations provided an indication of the responsiveness of the design models to changes in the parametric factors. Second, the analysis of the smaller models provided trade-off policy concerning the computation time involved, the size of the problems and the number of iterations till solution. Third, any direct comparison of the Goal Programming

results with those of more conventional methods has to be performed on a small scale as most conventional methods have been unable to consider the larger problems that have been resolved through Goal Programming.

The results of this three stage experiment in planar array design may be summarized as follows:

1. The patterns achieved compare quite favorably with the results obtained by other conventional approaches. This is an impressive achievement in view of the much more extensive and costly nature of the research done using conventional methods.
2. When compared with more traditional optimization methods, the Goal Programming method has achieved much better solution times in relatively small problems. If the traditional methods were extended to handle larger, more complex problems, this advantage should escalate in favor of Goal Programming.
3. Although Goal Programming can solve large problems (12 x 12 array formulations involved 52 variables and up to 193 objectives), it is suggested that those using the technique begin with a relatively simple problem in order to become acquainted with the technique and its options. This reduces the computation time and costs in the developmental stages of the experiment.

4.2 Design Specifications

Array patterns are often classified according to their shape. Two shapes often sought in practice are circular beam patterns and rectangular patterns. The circular beam attempted here was to have a 30° diameter, i.e., the 3 dB beamwidth about the beam axis. The rectangular beam, a more difficult pattern to realize in practice, attempted to achieve 72° x 36° dimensions. Both of these design problems were to obtain the specified circular or rectangular beam with uniform main beam response (within 3 dB if possible) and low response elsewhere (20 dB below that of the main beam). The model formulation for the circular pattern as such was presented in Chapter III.

4.3 Beam 1: A Circular Array Pattern

The amplitude and phasing ascertained through the nonlinear Goal Programming approach to achieve a 30° diameter circular beam are supplied in Figure 4.1. (Actually, the amplitude coefficients that were achieved through Goal Programming have been normalized to unity. This shading process resulted in a gain reduction of fifteen decibels at boresight.) The two-dimensional contour map in Figure 4.2 represents the radiation of the transducer array as a function of direction (θ,ϕ). The response, measured in negative decibels, does indeed provide a 30° diameter circular beam which meets or surpasses the necessary side lobe dropoff throughout most of the auxiliary lobes. Figure 4.3 is the three-dimensional representation of the radiation of the transducer array with these characteristics. Although the perspective of this plot is not optimal, the presence of the circular beam is still apparent.

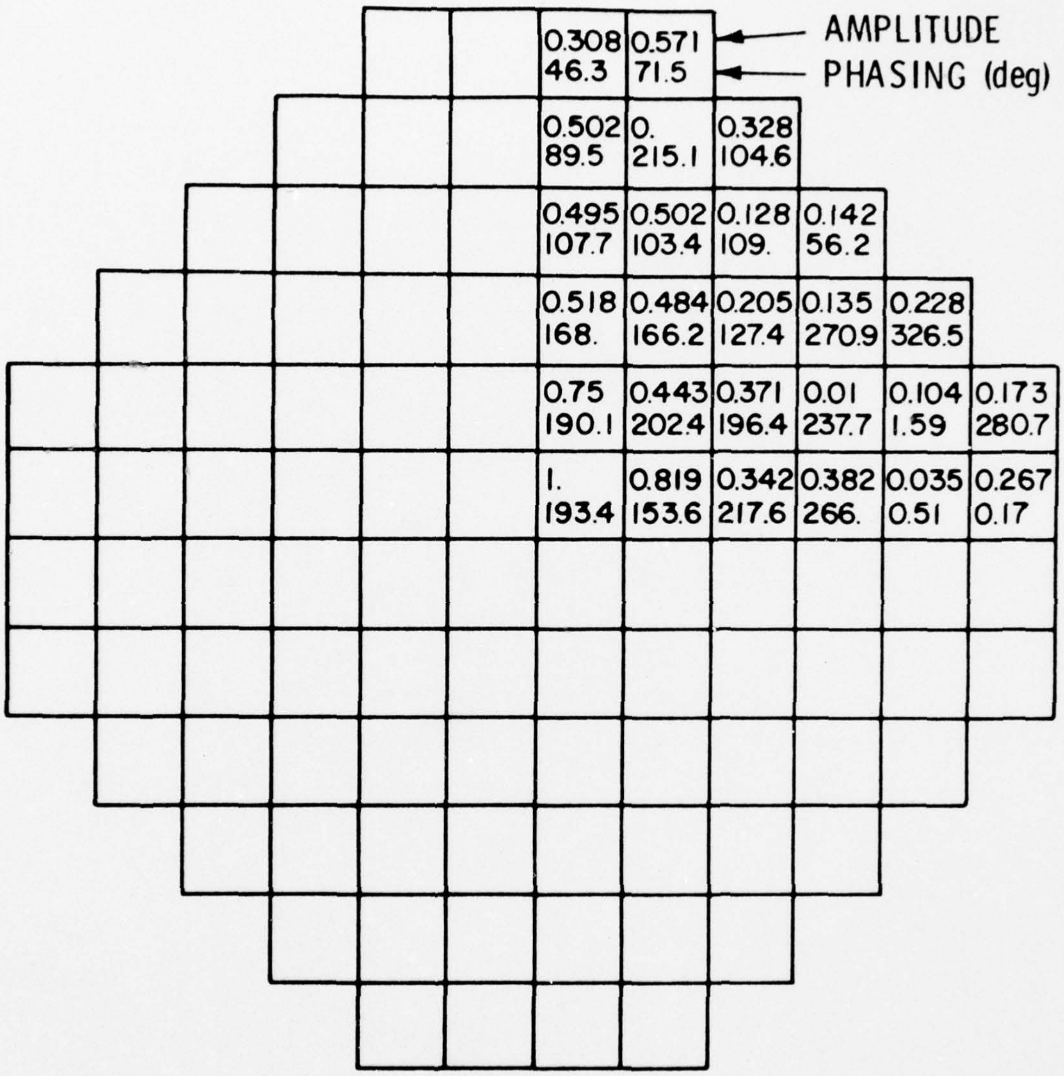

Figure 4.1. Amplitude and Phase Shading Coefficients for a Circular Beam

MAP OF ARRAY OUTPUT LEVEL

12 X 12 ARRAY --- 30 X 30 PATTERN.
WAVELENGTH = 1.0000
TOTAL NUMBER OF SOURCES = 104
NORMALIZING FACTOR = 18.45 OR 25.32 DB

PHI - DEGREES

THETA	0		10		20		30		40		50		60		70		80		90		100		110		120		130		140		150		160		170		180
0.00	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
3.00	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47
6.00	36	36	36	36	36	36	36	36	36	36	36	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	36	36	36	36	36	36	36	36	36	36	36
9.00	30	30	30	30	30	30	30	29	29	29	29	29	29	29	28	28	28	28	28	28	28	28	28	29	29	29	29	29	29	29	30	30	30	30	30	30	30
12.00	27	27	27	27	27	27	26	26	25	25	25	24	24	24	24	24	23	23	23	23	23	24	24	24	24	24	25	25	25	26	26	27	27	27	27	27	27
15.00	27	27	27	26	26	25	25	24	23	23	22	22	21	21	20	20	20	20	20	20	20	20	20	21	21	22	22	23	23	24	25	25	26	26	27	27	27
18.00	29	29	29	28	27	26	25	24	23	22	21	20	19	18	18	18	17	17	17	17	17	18	18	18	19	20	21	22	23	24	25	26	27	28	29	29	29
21.00	36	35	35	33	32	30	28	26	24	22	21	19	18	17	16	16	15	15	15	15	15	16	16	17	18	19	21	22	24	26	28	30	32	33	35	35	36
24.00	36	36	37	38	38	38	35	31	27	24	22	20	18	17	16	15	15	14	14	14	15	15	16	17	18	20	22	24	27	31	35	38	38	38	37	36	36
27.00	30	30	30	30	31	33	36	40	36	29	25	22	19	17	16	15	15	14	14	14	15	15	16	17	19	22	25	29	36	40	36	33	31	30	30	30	30
30.00	26	26	26	26	26	26	27	30	37	44	31	25	22	19	18	16	16	16	15	16	16	16	18	19	22	25	31	44	37	30	27	26	26	26	26	26	26
33.00	24	23	23	22	22	21	22	23	27	34	36	28	24	22	20	20	19	19	19	19	19	20	20	22	24	28	36	34	27	23	22	21	22	22	23	23	24
36.00	21	20	20	19	18	17	17	18	20	25	34	27	23	22	22	23	26	28	29	28	26	23	22	22	23	27	34	25	20	18	17	17	18	19	20	20	21
39.00	18	18	17	16	15	15	14	15	16	19	27	28	22	19	20	22	25	27	26	27	25	22	20	19	22	28	27	19	16	15	14	15	15	16	17	18	18
42.00	16	15	15	14	14	13	12	12	13	15	21	34	22	18	18	19	20	19	19	19	20	19	18	18	22	34	21	15	13	12	12	13	14	14	15	15	16
45.00	14	14	14	13	13	12	12	11	12	13	17	30	24	18	17	18	19	18	17	18	19	18	17	18	24	30	17	13	12	11	12	12	13	13	14	14	14
48.00	14	14	14	14	13	13	12	12	12	13	16	24	28	20	18	19	20	18	17	18	20	19	18	20	28	24	16	13	12	12	12	13	13	14	14	14	14
51.00	16	16	16	16	15	15	14	14	14	14	17	23	28	21	20	20	21	20	19	20	21	20	20	21	28	23	17	14	14	14	14	15	15	16	16	16	16
54.00	22	21	21	20	19	18	17	17	17	18	19	24	28	22	21	21	21	20	19	20	21	21	21	22	26	24	19	18	17	17	17	18	19	20	21	21	22
57.00	47	43	36	29	25	21	19	19	21	23	24	25	25	22	22	22	20	18	17	18	20	22	22	22	25	25	24	23	21	19	19	21	25	29	36	43	47
60.00	25	25	26	28	29	26	22	20	21	25	26	26	25	22	21	22	19	16	14	16	19	22	21	22	25	26	26	25	21	20	22	26	29	28	26	25	25
63.00	21	21	22	23	25	33	31	24	22	22	22	23	27	21	18	19	17	14	13	14	17	19	18	21	27	23	22	22	22	24	31	33	25	23	22	21	21
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69.00	22	22	21	21	21	22	21	16	15	14	14	16	25	25	19	23	37	28	25	28	37	23	19	25	25	16	14	16	15	18	21	22	21	21	21	22	22
72.00	26	26	24	23	23	23	21	17	14	13	14	16	22	36	30	23	13	10	9	10	13	23	30	36	22	16	14	13	14	17	21	23	23	23	24	26	26
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78.00	36	35	32	29	29	32	37	23	18	16	17	18	22	26	17	8	4	2	1	2	4	8	17	26	22	18	17	16	18	23	37	32	29	29	32	35	36
81.00	39	37	33	29	27	27	35	28	20	18	18	19	23	26	15	7	3	2	1	2	3	7	15	26	23	19	18	18	20	28	35	27	27	29	33	37	39
84.00	45	41	34	29	26	25	29	35	22	18	17	17	19	26	15	5	2	2	2	2	2	5	15	26	19	17	17	18	22	35	29	25	26	29	34	41	45
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141.00	18	18	17	16	15	15	14	15	16	19	27	28	22	19	20	22	25	27	26	27	25	22	20	19	22	28	27	19	16	15	14	15	15	16	17	18	18
144.00	21	20	20	19	18	17	17	18	20	25	34	27	23	22	22	23	26	28	29	28	26	23	22	22	23	27	34	25	20	18	17	17	18	19	20	20	21
147.00	24	23	23	22	22	21	22	23	27	34	36	28	24	22	20	20	19	19	19	19	19	20	20	22	24	28	36	34	27	23	22	21	22	22	23	23	24
150.00	26	26	26	26	26	26	27	30	37	44	31	25	22	19	18	16	16	16	15	16	16	16	18	19	22	25	31	44	37	30	27	26	26	26	26	26	26
153.00	30	30	30	30	31	33	36	40	36	29	25	22	19	17	16	15	15	14	14	14	15	15	16	17	19	22	25	29	36	40	36	33	31	30	30	30	30
156.00	36	36	37	38	38	38	35	31	27	24	22	20	18	17	16	15	15	14	14	14	15	15	16	17	18	20	22	24	27	31	35	38	38	38	37	36	36
159.00	36	35	35	33	32	30	28	26	24	22	21	19	18	17	16	16	15	15	15	15	15	16	16	17	18	19	21	22	24	26	28	30	32	33	35	35	36
162.00	29	29	29	28	27	26	25	24	23	22	21	20	19	18	18	18	17	17	17	17	17	18	18	18	19	20	21	22	23	24	25	26	27	28	29	29	29
165.00	27	27	27	26	26	25	25	24	23	23	22	22	21	21	20	20	20	20	20	20	20	20	20	21	21	22	22	23	23	24	25	25	26	26	27	27	27
168.00	27	27	27	27	27	27	26	26	25	25	25	24	24	24	24	24	23	23	23	23	23	24	24	24	24	24	25	25	25	26	26	27	27	27	27	27	27
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174.00	36	36	36	36	36	36	36	36	36	36	36	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	36	36	36	36	36	36	36	36	36	36	36
177.00	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47

THETA IS THE ANGLE OF INCIDENCE
PHI IS THE NULL ANGLE MEASURED FROM THE X AXIS
ALL NUMBERS ARE NEGATIVE DECIBELS

NOTE LABELLING OF THE PHI SCALE IS FOR ALTERNATE COLUMNS AND IS
TRUNCATED TO INTEGER MODE BECAUSE OF LIMITED SPACE
THE FIRST COLUMN IS FOR PHI = 0.00
THE INCREMENT BETWEEN COLUMNS = 5.00

Figure 4.2. Contour Map of the Circular Beam Response Pattern

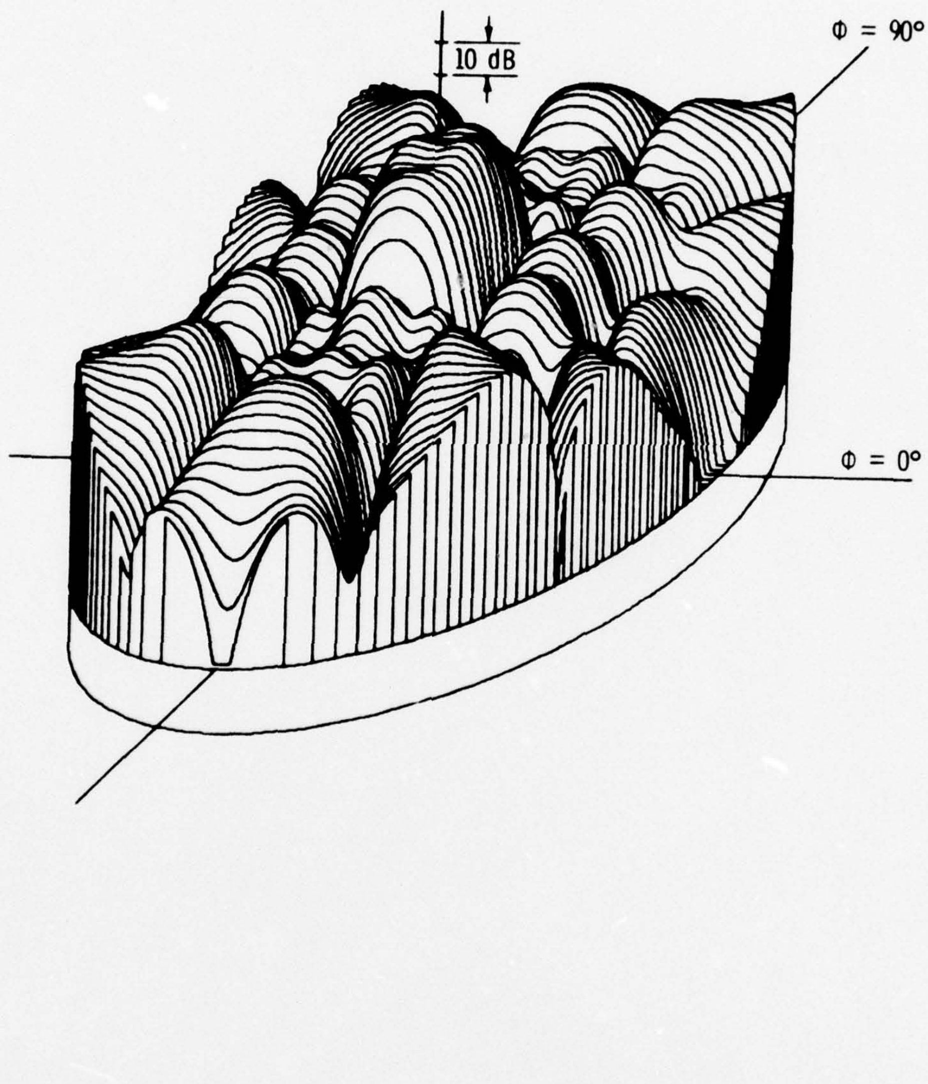

Figure 4.3. Three-Dimensional Plot of the Circular Beam Response Pattern

4.4 Beam 2: A Rectangular Array Pattern

Figure 4.4 provides the amplitude and phase coefficients which Goal Programming found to best satisfy the 72° x 36° rectangular specifications. (Here, again, the "optimal" amplitude coefficients were normalized to unity. In this case, the shading process reduced the gain by twenty-two decibels at boresight.) The contour map of the response pattern thus achieved is found in Figure 4.5. The only deviations from the desired response levels are the slight violations found at the corners of the main beam and the side lobe levels immediately adjoining the main beam. These violations are minor as can be seen from the three-dimensional plot of the radiation pattern in Figure 4.6.

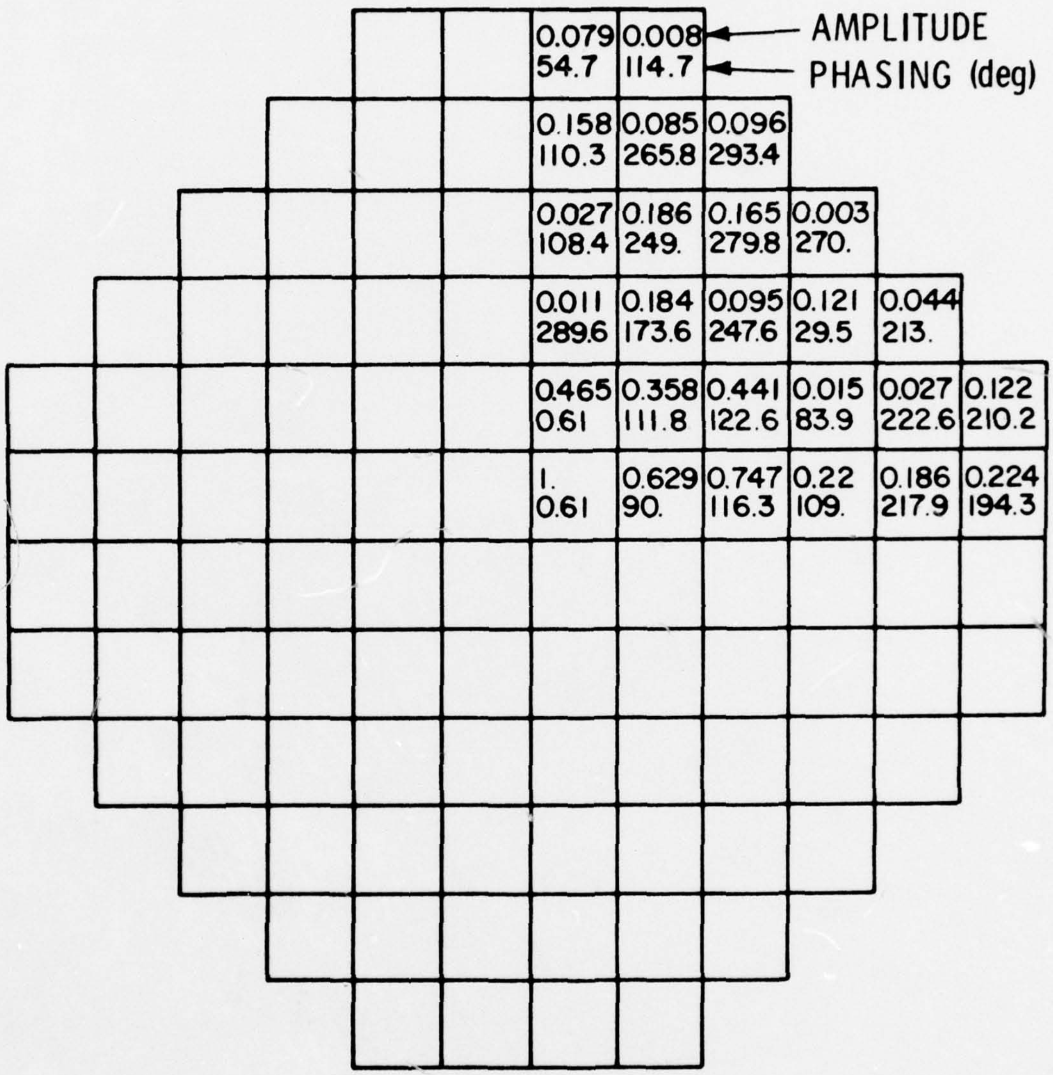

Figure 4.4. Amplitude and Phase Shading Coefficients for the Rectangular Beam

MAP OF ARRAY OUTPUT LEVEL

12 X 12 ARRAY --- 72 X 36.
WAVELENGTH = 1.0000
TOTAL NUMBER OF SOURCES = 104
NORMALIZING FACTOR = 0.24 DB 18.32 DB

PHI - DEGREES

THETA	0		10		20		30		40		50		60		70		80		90		100		110		120		130		140		150		160		170		180
0.00	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
3.00	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61
6.00	47	47	47	48	48	48	48	48	48	48	48	48	49	49	49	49	49	49	49	49	49	49	49	49	49	48	48	48	48	48	48	48	48	48	47	47	47
9.00	39	39	39	39	39	39	40	40	40	40	41	41	41	41	42	42	42	42	42	42	42	42	42	41	41	41	41	40	40	40	40	39	39	39	39	39	39
12.00	33	33	33	33	33	33	33	34	34	34	35	35	36	36	36	37	37	37	37	37	37	37	36	36	36	35	35	34	34	34	33	33	33	33	33	33	33
15.00	28	28	28	28	28	28	28	29	29	30	30	31	31	32	32	33	33	33	33	33	33	33	32	32	31	31	30	30	29	29	28	28	28	28	28	28	28
18.00	24	24	24	24	24	25	25	25	25	26	26	27	28	28	29	30	30	30	30	30	30	30	29	28	28	27	26	26	25	25	25	25	24	24	24	24	24
21.00	22	22	22	22	22	22	22	22	23	23	23	24	25	26	26	27	28	28	28	28	28	27	26	26	25	24	23	23	23	22	22	22	22	22	22	22	22
24.00	21	21	20	20	20	20	20	20	21	21	21	22	23	24	25	26	26	26	27	26	26	26	25	24	23	22	21	21	21	20	20	20	20	20	20	21	21
27.00	21	21	21	20	20	20	19	19	19	20	20	21	21	22	23	24	25	25	25	25	25	24	23	22	21	21	20	20	19	19	19	20	20	20	21	21	21
30.00	23	23	23	22	21	21	20	20	19	19	19	20	21	22	23	24	25	25	25	25	25	24	23	22	21	20	19	19	19	20	20	21	21	22	23	23	23
33.00	27	27	26	26	25	24	22	21	20	20	20	20	21	23	24	25	25	25	24	25	25	25	24	23	21	20	20	20	20	21	22	24	25	26	26	27	27
36.00	26	26	26	27	28	29	27	25	23	22	22	22	23	25	27	29	28	25	24	25	28	29	27	25	23	22	22	22	23	25	27	29	28	27	26	26	26
39.00	23	23	23	23	26	28	29	32	30	27	25	25	27	30	34	42	32	27	25	27	32	42	34	30	27	25	25	27	30	32	29	26	24	23	23	23	23
42.00	22	22	22	22	22	23	25	28	37	36	31	30	31	32	30	31	32	29	28	29	32	31	30	32	31	30	31	36	37	28	25	23	22	22	22	22	22
45.00	23	23	23	22	22	22	22	24	29	42	37	32	30	26	24	23	24	27	28	27	24	23	24	26	30	32	37	42	29	24	22	22	22	22	23	23	23
48.00	25	25	25	25	24	23	23	24	26	34	43	32	28	24	20	19	20	23	24	23	20	19	20	24	28	32	43	34	26	24	23	23	24	25	25	25	25
51.00	24	24	25	27	29	28	25	24	25	30	39	35	30	23	18	17	18	20	21	20	18	17	18	23	30	35	39	30	25	24	25	28	29	27	25	24	24
54.00	24	24	25	28	33	35	28	25	24	25	29	33	34	26	18	15	16	19	21	19	16	15	18	26	34	33	29	25	24	25	28	35	33	28	25	24	24
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60.00	26	27	28	30	36	32	25	22	20	21	26	44	28	20	14	11	11	14	16	14	11	11	14	20	28	44	26	21	20	22	25	32	36	30	28	27	26
63.00	25	25	26	27	29	28	24	21	21	24	30	23	18	15	11	8	8	9	10	9	8	8	11	15	18	23	30	24	21	21	24	28	29	27	26	25	25
66.00	24	24	24	24	25	25	23	21	22	33	21	14	11	9	8	6	5	6	6	6	5	6	8	9	11	14	21	33	22	21	23	25	25	24	24	24	24
69.00	25	24	24	23	24	24	22	20	23	25	14	9	6	5	5	4	3	3	3	3	3	4	5	5	6	9	14	29	23	20	22	24	24	23	24	24	25
72.00	27	26	25	24	24	24	23	20	22	20	11	6	3	2	3	3	2	2	2	2	2	3	3	2	3	6	11	20	22	20	23	24	24	24	25	26	27
75.00	31	30	28	27	26	27	24	20	21	17	8	4	1	1	2	2	2	2	1	2	2	2	2	1	1	4	8	17	21	20	24	27	26	27	28	30	31
78.00	36	35	32	30	30	33	27	21	21	16	7	3	0	0	1	2	2	2	1	2	2	2	1	0	0	3	7	16	21	21	27	33	30	30	32	35	36
81.00	41	39	35	31	31	37	31	23	22	15	7	2	0	0	1	2	2	2	1	2	2	2	1	0	0	2	7	15	22	23	31	37	31	31	35	39	41
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87.00	57	45	35	29	26	24	25	25	23	13	5	2	1	1	2	2	1	2	2	2	1	2	2	1	1	2	5	13	23	25	25	24	26	29	35	45	57
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96.00	46	42	35	30	28	28	29	25	24	14	6	2	1	0	1	2	1	2	2	2	1	2	1	0	1	2	6	14	24	25	29	28	28	30	35	42	46
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111.00	25	24	24	23	24	24	22	20	23	25	14	9	6	5	5	4	3	3	3	3	3	4	5	5	6	9	14	29	23	20	22	24	24	23	24	24	25
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117.00	25	25	26	27	29	28	24	21	21	24	30	23	18	15	11	8	8	9	10	9	8	8	11	15	18	23	30	24	21	21	24	28	29	27	26	25	25
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126.00	24	24	25	28	33	35	28	25	24	25	29	33	34	26	18	15	16	19	21	19	16	15	18	26	34	33	29	25	24	25	28	35	33	28	25	24	24
129.00	24	24	25	27	29	28	25	24	25	30	39	35	30	23	18	17	18	20	21	20	18	17	18	23	30	35	39	30	25	24	25	28	29	27	25	24	24
132.00	25	25	25	25	24	23	23	24	26	34	43	32	28	24	20	19	20	23	24	23	20	19	20	24	28	32	43	34	26	24	23	23	24	25	25	25	25
135.00	23	23	23	22	22	22	22	24	29	42	37	32	30	26	24	23	24	27	28	27	24	23	24	26	30	32	37	42	29	24	22	22	22	22	23	23	23
138.00	22	22	22	22	22	23	25	28	37	36	31	30	31	32	30	31	32	29	28	29	32	31	30	32	31	30	31	36	37	28	25	23	22	22	22	22	22
141.00	23	23	23	23	26	28	29	32	30	27	25	25	27	30	34	42	32	27	25	27	32	42	34	30	27	25	25	27	30	32	29	26	24	23	23	23	23
144.00	26	26	26	27	28	29	27	25	23	22	22	22	23	25	27	29	28	25	24	25	28	29	27	25	23	22	22	22	23	25	27	29	28	27	26	26	26
147.00	27	27	26	26	25	24	22	21	20	20	20	20	21	23	24	25	25	25	24	25	25	25	24	23	21	20	20	20	20	21	22	24	25	26	26	27	27
150.00	23	23	23	22	21	21	20	20	19	19	19	20	21	22	23	24	25	25	25	25	25	24	23	22	21	20	19	19	19	20	20	21	21	22	23	23	23
153.00	21	21	21	20	20	20	19	19	19	20	20	21	21	22	23	24	25	25	25	25	25	24	23	22	21	21	20	20	19	19	19	20	20	20	21	21	21
156.00	21	21	20	20	20	20	20	20	21	21	21	22	23	24	25	26	26	26	27	26	26	26	25	24	23	22	21	21	21	20	20	20	20	20	20	21	21
159.00	22	22	22	22	22	22	22	22	23	23	23	24	25	26	26	27	28	28	28	28	28	27	26	26	25	24	23	23	23	22	22	22	22	22	22	22	22
162.00	24	24	24	24	24	25	25	25	25	26	26	27	28	28	29	30	30	30	30	30	30	30	29	28	28	27	26	26	25	25	25	25	24	24	24	24	24
165.00	28	28	28	28	28	28	28	29	29	30	30	31	31	32	32	33	33	33	33	33	33	33	32	32	31	31	30	30	29	29	28	28	28	28	28	28	28
168.00	33	33	33	33	33	33	33	34	34	34	35	35	36	36	36	37	37	37	37	37	37	37	36	36	36	35	35	34	34	34	33	33	33	33	33	33	33
171.00	39	39	39	39	39	39	40	40	40	40	41	41	41	41	42	42	42	42	42	42	42	42	42	41	41	41	41	40	40	40	40	39	39	39	39	39	39
174.00	47	47	47	48	48	48	48	48	48	48	48	48	49	49	49	49	49	49	49	49	49	49	49	49	49	48	48	48	48	48	48	48	48	48	47	47	47
177.00	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61

THETA IS THE ANGLE OF INCIDENCE
PHI IS THE ROLL ANGLE MEASURED FROM THE X AXIS
ALL NUMBERS ARE NEGATIVE DECIBELS

NOTE LABELLING OF THE PHI SCALE IS FOR ALTERNATE COLUMNS AND IS TRUNCATED TO INTEGER MODE BECAUSE OF LIMITED SPACE
THE FIRST COLUMN IS FOR PHI = 0.00
THE INCREMENT BETWEEN COLUMNS = 5.00

Figure 4.5. Contour Map of the Rectangular Response Pattern

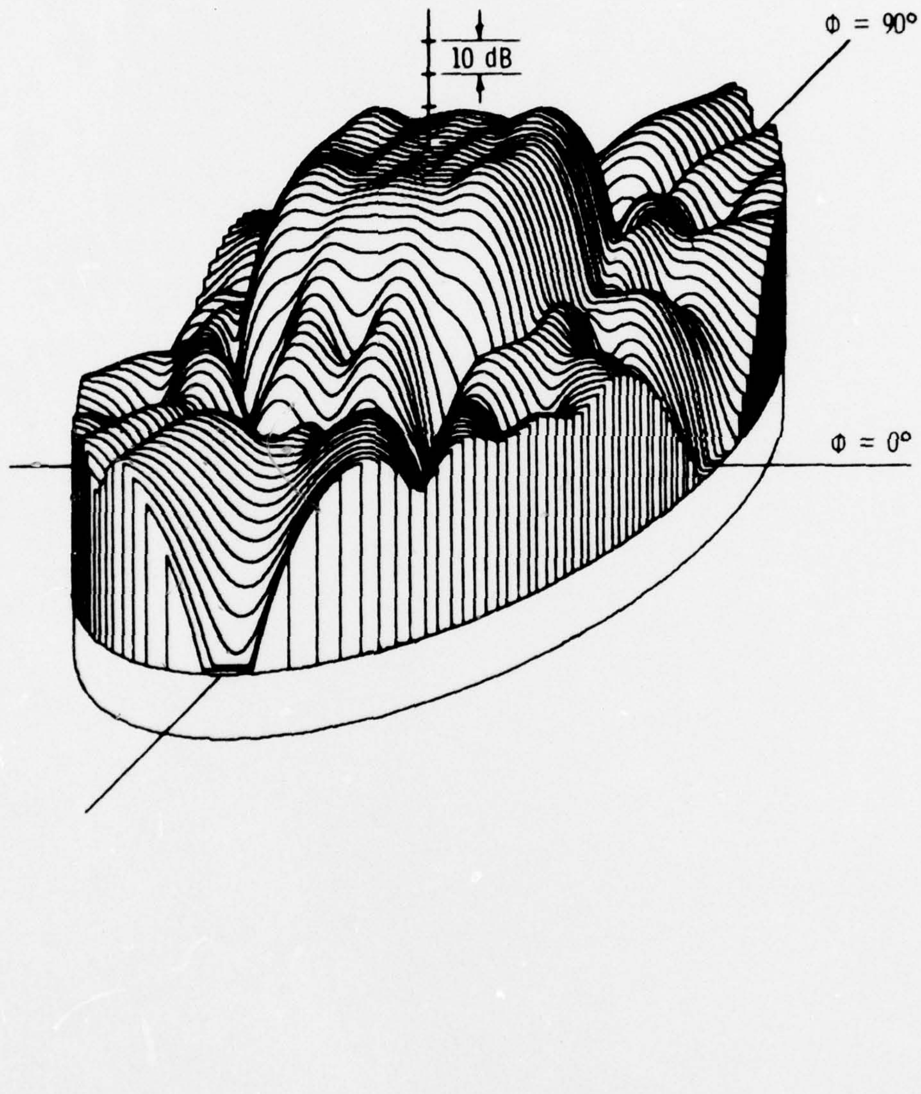

Figure 4.6. Three-Dimensional Plot of the Rectangular Response Pattern

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Summary

The broad underlying objective of this study was to demonstrate the Goal Programming approach in the analysis and solution of multi-criteria system design problems.

The particular application studied has been the design of planar arrays in order to achieve specific response objectives. The formulation and achievement of the 12 x 12 planar array designs outlined within this thesis provide an indication of the typical response objectives encountered and the ability of Goal Programming to satisfy the multiple, conflicting goals of such designs.

The specific objectives of this thesis, as given in Chapter I, were:

1. Develop a mathematical model which describes the multicriteria array design problem via a set of mathematical statements and extend this model to the goal programming formulation.

2. Achieve two specific acoustic array designs using the Goal Programming approach.

3. Discuss the versatility of Goal Programming in array design relative to other tools used in such designs.

4. Through the successful nature of the results achieved and the straightforward nature of the problem formulation, encourage the use of Goal Programming in resolution of even more complex array design problems.

The first objective is responded to in Chapter III. The chapter on empirical results, Chapter IV, provides the results of the second objective. Chapters II and IV combine to satisfy the third objective. Finally, it is hoped that the achievement of the fourth objective will be encouraged by the accomplishment of the first three.

In support of the fourth objective, a summary of the process necessary to use the computerized Goal Programming approach to array design follows:

1. The development of a preliminary mathematical model which essentially describes the multicriteria design problem via a set of mathematical statements.
2. The extension of the preliminary mathematical model to the goal programming formulation. This involves the addition of deviation variables in the objective functions and the statement of the "achievement function" in terms of the deviation variables.
3. The development of the subroutine (YVALUE) which defines the objectives of the system design in the FORTRAN computing language. (A listing of that subroutine for the design of the circular beam presented herein may be found in Appendix C.)
4. The solution of the design problem using subroutine YVALUE in conjunction with the nonlinear Goal Programming code as listed and documented in the appendix of Reference 3. The use of this code is facilitated by the user's guide presented in

Reference 3, as well as the analysis of the procedure provided in Appendix B of this thesis.

5.2 Conclusions

It is felt that the basic nature of most real design problems should strongly emphasize the need for Goal Programming with its capacity to resolve multiple, conflicting design objectives. By the nature of this capacity, the Goal Programming model has no constraint functions and thus more efficient optimization techniques are employed in its solution.

Another highly favorable characteristic of the Goal Programming technique to be reiterated in conclusion is its straightforward extension to other complex multicriteria design problems. It is hoped that this easy transition to more complex problems coupled with the success of this research endeavor will stimulate the use of Goal Programming in further system designs.

APPENDIX A

OUTPUT FROM AN ARRAY CONSISTING OF SETS OF SYMMETRICALLY LOCATED ELEMENTS

A.1 The Derivation of the Power Response Objectives of a Symmetric Planar Array

Consider an element of a planar transducer array located at a point whose polar coordinates are (r,δ) in the X-Y plane (Figure 4, see text). The output from a signal arriving from a direction (θ,ϕ) is:

$$P_{r,\delta} = Ae^{j(\omega t + u)}$$

$$= Ae^{j\omega t}(\cos u + j \sin u) \quad , \qquad (A.1)$$

where:

$$u = kr \sin\theta \cos(\phi - \delta) + \alpha$$

$$= kx \sin\theta \cos\phi + ky \sin\theta \sin\phi + \alpha \qquad (A.2)$$

and

A is the shading coefficient

k is the wave-number $2\pi/\lambda$

and α is the element phasing.

The output of an array of M such elements is:

$$P_{tot} = \sum_{m=1}^{M} A_m e^{j\omega t}(\cos u_m + j \sin u_m) \quad . \qquad (A.3)$$

The term $e^{j\omega t}$ merely indicates the frequency dependence, and is dropped when the equation is used for the analysis of directional response.

Taking advantage of the symmetry inherent in the problem, the expression for the total output can be further simplified. For a pair of elements at opposite ends of a diameter through the origin, i.e., at (r,δ) and $(-r,\delta)$ with the same phase shift α applied, we have:

$$\begin{aligned} P = P_+ + P_- &= A[\cos(u' + \alpha) + \cos(\alpha - u') \\ &\qquad + j \sin(u' + \alpha) + \sin(\alpha - u')] \\ &= 2 A \cos u'(\cos \alpha + j \sin \alpha) \quad , \qquad \text{(A.4)} \end{aligned}$$

where $u' = u - \alpha$.

Thus, the output of an array of M such pairs may be written:

$$P_{\Sigma tot} = 2 \sum_{m=1}^{M} A_m \cos u'_m(\cos \alpha_m + j \sin \alpha_m) \quad . \qquad \text{(A.5)}$$

For M sets of four elements in quadrantal symmetry [whose cartesian coordinates may be written (x_m,y_m) , $(-x_m,y_m)$, $(x_m,-y_m)$, $(-x_m,-y_m)$] all with the same coefficient A_m and phase α_m , we may incorporate that symmetry to obtain:

$$\begin{aligned} P_{\Sigma Q} = 4 \sum^{m} A_m(\cos \alpha_m + j \sin \alpha_m) &\cos(k x_m \sin \theta \cos \phi) \\ &\cos(k y_m \sin \theta \sin \phi) \quad . \end{aligned}$$

If we are not interested in the phase, we may write for the power response:

$$\begin{aligned} W_{\Sigma Q} = 16 \Bigg[&\left(\sum_{m=1}^{M} A_m \cos \alpha_m \cos U_m \cos V_m\right)^2 \\ &+ \left(\sum_{m=1}^{M} A_m \sin \alpha_m \cos U_m \cos V_m\right)^2 \Bigg] \quad , \qquad \text{(A.6)} \end{aligned}$$

where:

$$U_m = k\, x_m \sin\theta \cos\phi$$

and

$$V_m = k\, y_m \sin\theta \sin\phi \quad .$$

APPENDIX B

THE EXPERIMENT

B.1 Introduction to the Modified Hooke and Jeeves Algorithm

In order to describe the procedure used to achieve a "satisfactory" design, it is necessary to further elaborate on Ignizio's extension of the pattern search method of Hooke and Jeeves employed by the nonlinear Goal Programming algorithm to solve the multiple objective nonlinear model [3].

Simply stated, pattern search begins with an initial base point, provided as input to the algorithm, and perturbs about this point along all the coordinates. In this manner, it searches along all dimensions accelerating the search if previous searches have been successful and decreasing otherwise. Input to the algorithm includes the initial step sizes for each dimension, an acceleration factor to be used upon success, and a step reduction factor to be employed when search in all directions fails to improve the solution. Thus, pattern search begins at the specified initial point and searches along each axis at the step size given. If it succeeds in any direction, search advances from this successful point along the next coordinate. After searching along each dimension, the solution vector achieved as the result of the search is tested to see if it gives a preferred achievement vector. If the new solution has an improved achievement vector, $\bar{a}$, the resultant point becomes the new base point. The search is then accelerated by renewing

the search at a trial point found at the end of a vector from the old base point through the new base point at a length of the acceleration factor times the distance from the old and new base points. Therefore, when the search succeeds in a given direction, pattern search assumes that the search is heading in the right direction and accelerates along this path. If search about a point fails to improve the achievement vector values, pattern search returns to the most recent base point (the most successful solution to-date) and perturbs around it at reduced distances of the reduction factor times the step size. Thus, failure indicates that the optimal solution does not lie in this direction or has been overstepped and pattern search attempts to backtrack and reduce the step size of the search. Any subsequent failures to improve the solution are met with further reduction of the step sizes by further multiplication by the reduction factor.

Termination of the search algorithm will occur when the search has achieved a local optimum which may be the global value. In order to facilitate recognition of this optimum and avoid the continuation of the algorithm when it is only improving the base point by a very small increment (often a computational round-off factor), the algorithm input specifies a termination factor. Whenever two sequential iterations in the search lead to achievement values that differ by no more than this factor, the search is completed. It was necessary to implement another rule of termination as, oftentimes, the pattern search algorithm can continue advancing the solution by a small design improvement at each iteration. This improvement when weighed against the computation time involved in each iteration may not be sufficient to warrant continuation of the search.

The rule provided is to terminate the search algorithm after a prespecified number of iterations. This is another factor which is provided by the user for the Goal Programming code. Another safeguard provides against pattern cycling upon successive failures and is a user designated factor which dictates the maximum number of increment reductions.

With this brief introduction to the pattern search algorithm, several factors have been brought to light. Input to the algorithm must provide the initial starting point, the initial search step sizes, the acceleration factor, the step reduction factor, the maximum number of search patterns, and the maximum number of increment reductions.

B.2 Algorithm Starting Points

To compensate for the complex, multimodal nature of antenna pattern response functions, each design problem formulation has been subjected to several solution attempts using different starting points. It is apparent that the closer the starting point is to the optimal, the more quickly the algorithm is likely to converge. Keeping this point in mind, while iteratively repeating a specific array design problem with any of a number of parametric changes to improve upon the pattern achieved, the antenna design arrived at in the prior iteration was used as the initial point for continued analysis. In an attempt to save computation time in larger problem formulations, it is suggested that many randomly generated starting points be evaluated with the "best" of these employed as input to the algorithm.

B.3 Algorithm Initial Step Sizes

The initial decision on step sizes is an intuitive one. If the optimal solution is thought to be relatively close to the specified starting point, the search should begin at small intervals. The choice of the "best" step sizes is at first an iterative procedure, determining those step sizes which consistently overstep or underachieve the increment necessary to achieve the optimal. Use of the appropriate step sizes minimizes the computation time necessary by minimizing the number of step reductions or accelerations necessary. To elaborate, if the initial step size is too large, the algorithm may have to undergo a series of reduction cycles before successfully moving to an improved point in a given dimension.

B.4 Algorithm Acceleration and Reduction Factors

The choice of acceleration and reduction factors is also based on exposure to a number of different values. The optimal value for both are characteristic of the problem at hand as the factors are dependent upon the functions to be optimized. Any inadequacies in the acceleration or reduction factors are compensated for by the iterative process of acceleration and reduction, but again, the computation time consumed by this process can be alleviated by adjusting these values.

B.5 Achievement Function Tolerances

The achievement function tolerances are also dependent upon the characteristic values of the achievement function vector. In some design problems, a difference of 0.01 in achievement function values is significant, whereas, in others this is not an accurate or valuable

measure of improvement. The larger the values of the tolerances, the tighter the default constraint on achievement function improvements; thus, rapid termination of the algorithm can be brought about by setting high tolerances. This measure is of use when computation time becomes a serious consideration due to the size of the problem. By means of the specification of the maximum number of search patterns and the maximum number of increment reductions, two additional options are available to regulate the computation time.

B.6 The Number of Response Objectives

The initial decision concerning the number of response functions to include as the design objectives (functions of θ and ϕ) is also basically an intuitive one. Initially, the number of response objectives should be determined on the basis of any previous knowledge or intuition concerning the ease of attaining the desired pattern. If the pattern is naturally "easy" to attain, the number of response functions defined in the objectives may be few. If the desired response is an imposing one, numerous objectives may have to be included in the model in order to control response inadequacies that would occur within sizeable unconstrained regions. It is best to begin with a conservative estimate of the number of objectives necessary as this allows the potential to see where, if at all, the response pattern is naturally weak.

In choosing the θ,ϕ values to be included in the response objectives, it is best to begin with equally spaced radiation levels as unequal spacing may apply an unnecessary amount of pressure on the more closely constrained area of response at the expense of the radiation levels in the region with few constraints.

If upon inspection of the pattern achieved by this initial attempt it is found that an unconstrained area of the response fails to meet the desired level and is possibly even affecting the response within the constrained region, the "grid" of the objectives in this area should be increased. This step should be taken with the understanding that the radiation level or response may merely shift, although the effects may be diffused into a larger area causing a problem not nearly as drastic as the one it alleviated. If many undesirable response levels occur within the unconstrained regions, the logical step is to tighten the grid of design objectives. In other words, an overall weak pattern due to the response in unconstrained response regions may generally be alleviated by increasing the radiation levels (functions of θ,ϕ) included in the objectives of the model.

B.7 Weighting Factors

Unless some insight is available concerning the response difficulties of a particular design before building the model, it is best initially to give equal weight to the design objectives of both the main beam and side lobes. The pattern arrived at when the nonlinear Goal Programming algorithm attempts to fulfill these equally weighted objectives will give some indication of the inherent difficulties in the design of the desired pattern. If the pattern achieves or exceeds the design objectives on the side lobes while the main beam or regions within the beam fail to meet its desired levels, the model should be restated with additional weights on the main beam objectives, forcing the algorithm to place additional importance on the response levels at the designated areas of the beam. The decision as to the magnitude of these weights is gained through actual experience,

with numerical values determined by the degree of difficulty characteristic of the region to be weighted.

As was previously stated, the arrival of the "correct" weighting factors for a given design is often an iterative process with the completion of the process signaled upon arrival at a "satisfactory" antenna array design.

B.8 The Response Levels

When establishing the levels to be achieved by the design objectives, the difficulty or impossibility of achieving a particular pattern at the levels specified may not be known. It may be found that unrealistic goals have been set, e.g., the side lobes cannot maintain an overall response level as far removed from that of the main beam as specified. A judgment of this nature should not be made without evaluating the results of a number of starting points, a range of weighting factor configurations, and a sufficient number of objectives. If it is found that unrealistic goals have been set, it is possible to iteratively reformulate the model and establish an upper bound on the response levels that can be achieved for a particular response pattern.

The process of adjustment can begin by bringing down the specifications for an entire region to levels easily achieved by most of the region. The algorithm should then focus its attention on the achievement of these adjusted levels in the trouble-spots as all other radiation levels have achieved the goals set. If the pattern responds, then the specifications can be iteratively increased seeking the upper limit on the response that can be achieved.

This concludes the analysis of the procedure used in the design of planar antenna arrays with the computerized nonlinear Goal Programming algorithm.

APPENDIX C

SUBROUTINE YVALUE TO PROVIDE THE MATHEMATICAL MODEL FOR THE 12 x 12 ARRAY ACHIEVING CIRCULAR BEAM SPECIFICATIONS

```
      SUBROUTINE YVALUE
      COMMON/COMM07/Y
      COMMON/COMM02/X
      DIMENSION Y(360),X(250)
      DIMENSION C(26),D(26),P(7)
      DIMENSION S(26),Q(26)
      DATA C/.25,.75,1.25,1.75,2.25,2.75,.25,.75,1.25,1.75,2.25,2.75,.25
     1,.75,1.25,1.75,2.25,.25,.75,1.25,1.75,.25,.75,1.25,.25,.75/
      DATA D/.25,.25,.25,.25,.25,.25,.75,.75,.75,.75,.75,.75,1.25,1.25,1
     2.25,1.25,1.25,1.75,.175,1.75,1.75,2.25,2.25,2.25,2.75,2.75/
      DATA P/0.,.2618,.5235,.7852,1.047,1.3088,1.5705/
C-----------------------------------------------------------------------
C-----THIS SUBROUTINE PROVIDES THE OBJECTIVES OF THE SYSTEM DESIGN.
C-----C,D ARE THE CARTESIAN COORDINATES (IN WAVELENGTHS).
C-----THE VECTOR P CONTAINS ALL THE ANGLES OF PHI DEFINED IN THE MODEL
C-----(IN RADIANS).
C-----Y IS THE VECTOR OF DESIGN OBJECTIVES.
C-----X IS THE VECTOR OF AMPLITUDE AND PHASE COEFFICIENTS.
C-----X(1) THROUGH X(26) CONTAIN THE AMPLITUDE COEFFICIENTS.
C-----X(27) THROUGH X(52) CONTAIN THE PHASING OF EACH OF THE ARRAY
C-----ELEMENTS.
C-----------------------------------------------------------------------
C-----THE FOLLOWING LOOP ESTABLISHES THE RANGE ON THE PHASING AND THE
C-----LOWER RANGE ON THE AMPLITUDE.
C-----AMPLITUDE GREATER THAN OR EQUAL TO 0.
C-----ALL PHASING GREATER THAN OR EQUAL TO 0 DEGREES
C-----AND LESS THAN OR EQUAL TO 360 DEGREES.
C-----------------------------------------------------------------------
      DO 5 I=1,26
      M=I+26
      K=I+52
      Y(I)=X(I)
      Y(M)=X(M)
5     Y(K)=X(M)
```

```
C-----------------------------------------------------------------------
C-----THE FOLLOWING DEFINES THE MAIN BEAM DESIGN OBJECTIVES.
C-----THE PATTERN IS TO ACHIEVE A 30 DEGREE DIAMETER BEAM.
C-----------------------------------------------------------------------
      K=78
C-----------------------------------------------------------------------
C-----THIS LOOP ACCOUNTS FOR THE SEVEN ANGLES OF PHI CONSTRAINED.
C-----EACH VALUE IS HELD CONSTANT WHILE AN INTERNAL LOOP GENERATES ALL
C-----THE VALUES OF THETA AT WHICH THE MAIN BEAM RESPONSE IS MEASURED.
C-----------------------------------------------------------------------
      DO 10 M=1,7
C-----T REFERS TO THETA WHICH INITIALLY EQUALS 0 DEGREES (0 RADIANS).
      T=0.
C-----------------------------------------------------------------------
C-----THIS LOOP ACCOUNTS FOR THE FOUR CONSTRAINED ANGLES OF THETA
C-----CONTAINED IN THE MAIN BEAM (0,5,10;15 DEGREES) WHILE THE VALUE
C-----OF PHI REMAINS CONSTANT.
C-----------------------------------------------------------------------
      DO 15 L=1,4
      QM=0.
      SM=0.
C-----------------------------------------------------------------------
C-----THIS INNER LOOP COMPUTES AND SUMS THE CONTRIBUTION OF EACH ARRAY
C-----ELEMENT.
C-----------------------------------------------------------------------
      DO 20 I=1,26
      II=I+26
      R=COS(6.283*C(I)*SIN(T)*COS(P(M)))*COS(6.283*D(I)*SIN(T)*SIN(P(M))
     3)
      S(I)=X(I)*COS(X(II))*R
      Q(I)=X(I)*SIN(X(II))*R
      SM=SM+S(I)
      QM=QM+Q(I)
20    CONTINUE
```

```
C-----------------------------------------------------------------------
      K=K+1
C-----ZZ IS THE RESPONSE AS CALCULATED FOR EACH SET OF THETA AND PHI
C-----VALUES.
      ZZ=10*(ALOG10(16((SM**2)+(QM**2))))
      IF(M.GT.1.OR.L.GT.1) GO TO 18
C-----Z IS THE RESPONSE AT THETA = 0 RADIANS AND PHI = 0 RADIANS WHICH
C-----IS USED AS THE REFERENCE RESPONSE LEVEL.
      Z=ZZ
C-----Y(K) IS THE ACTUAL VALUE OF THE K-TH OBJECTIVE AT A GIVEN
C-----AMPLITUDE AND PHASING.
18    Y(K)=ZZ-Z
      T=T+.08725
15    CONTINUE
10    CONTINUE
C-----------------------------------------------------------------------
C-----THE FOLLOWING DEFINES THE SIDE LOBE DESIGN OBJECTIVES.
C-----------------------------------------------------------------------
C-----THIS LOOP ACCOUNTS FOR THE SEVEN ANGLES OF PHI CONSTRAINED.
C-----EACH IS HELD CONSTANT WITHIN THE LOOP WHILE AN INTERNAL LOOP
C-----GENERATES ALL THE VALUES OF THETA AT WHICH THE SIDE LOBE
C-----RESPONSE IS MEASURED.
C-----------------------------------------------------------------------
      DO 35 M=1,7
C-----------------------------------------------------------------------
C-----THETA IS INITIALIZED AT 15 DEGREES AND INCREMENTED BY 5 DEGREES
C-----WITHIN THE FOLLOWING LOOP, WHEREIN, THE RESPONSE IS MEASURED AT
C-----THETA = 20,25,30,...,90 DEGREES.
C-----------------------------------------------------------------------
      T=.26175
      DO 30 L=1,15
      T=T+.08725
      QM=0.
      SM=0.
```

```
C---------------------------------------------------------------------------
C-----THIS INNER LOOP COMPUTES AND SUMS THE CONTRIBUTION OF EACH ARRAY
C-----ELEMENT.
C---------------------------------------------------------------------------
      DO 25 I=1,26
      II=I+26
      R=COS(6.283*C(I)*SIN(T)*COS(P(M)))*COS(6.283*D(I)*SIN(T)*SIN(P(M))
     4)
      S(I)=X(I)*COS(X(II))*R
      Q(I)=X(I)*SIN(X(II))*R
      SM=SM+S(I)
      QM=QM+Q(I)
25    CONTINUE
C---------------------------------------------------------------------------
      K=K+1
C-----ZZ IS THE RESPONSE AS CALCULATED FOR EACH SET OF THETA AND PHI
C-----VALUES.
      ZZ=10*(ALOG10(16*((SM**2)+(QM**2))))
C-----Y(K) IS THE ACTUAL VALUE OF THE K-TH OBJECTIVE AT A GIVEN
C-----AMPLITUDE AND PHASING.
      Y(K)=Z-ZZ
30    CONTINUE
35    CONTINUE
      RETURN
      END
```

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